Instrumental Variables Algorithm for Modal Parameter Identification in Flutter Testing

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An instrumental variables algorithm for modal parameter identification is derived in the frequency domain, and an example of its use in aeroelasticity testing is given. Basically the algorithm fits a set of poles and zeros to the measured transfer function of a linear, time-invariant system. An instrumental variables estimate is similar to a least-squared-error estimate but without the bias of the latter for noisy data. The algorithm was implemented for on-line data reduction using a minicomputer-based analysis system, with less core and computation time requirements than the data acquisition process. With the instrumental variables algorithm, accurate and reliable stability estimates can be obtained from a reasonable length of data.

Introduction

A MAJOR task in the development of a new aircraft is the demonstration of satisfactory aeroelastic characteristics. Beyond the determination of the flutter boundary or establishing that the aircraft is flutter-free throughout its operating envelope, it also is desirable to obtain detailed dynamic information, both to determine directly specific characteristics such as the aircraft gust response and to provide data for the verification of mathematical models of the aircraft. Thus the flutter testing of an aircraft requires an accurate, efficient, and reliable method to measure the aeroelastic response and to obtain from the response the parameters defining the dynamic characteristics. The most important parameters are generally those defining the flutter-stability level, namely, the frequency and damping of the low-damped modes of the system. The state-of-the-art of dynamic stability testing in the aircraft industry has been reviewed in a number of surveys.[1-5] The problem usually involved is the flutter testing of the airplane wing or tail in flight. Some form of frequency-response procedure is common. There are many variations, but a typical procedure involves fast swept-sine excitation and digital analysis of the response (analog analysis is also still common), the frequency and damping being determined from the circle on the phase plane.[6] Another approach often used is to determine dynamic stability from the decaying transient response. Other measurement and analysis techniques have been developed for structural dynamics testing, but in such testing the principal data required are the frequencies and mode shapes, whereas in flutter testing damping is of primary concern.

Flutter testing involves two major tasks: measuring the response of the system, and then extracting the modal parameters from the response. A number of techniques for measuring the system response were examined in Ref. 7, including a consideration of their error statistics. It was concluded that the best accuracy can be obtained from the minimum amount of data by exciting the system with an external input and measuring the system transfer function. Flutter-testing methods, such as a fast sine-sweep measurement of the transfer function or the transient decay technique (perhaps using the moving block analysis in the data reduction), rely on a high signal-to-noise ratio to obtain an accurate response measurement. With clean data these methods can work well, but, as the noise increases, the accuracy of the flutter parameter determination rapidly degrades, especially the damping estimate. Often, the experimenter has no control over the noise level, and the system excitation cannot be increased substantially because of structural or another limitations, and so little can be done to control the relative error level. To improve the accuracy of the measurement of the system response, the data must be averaged. Autospectral analysis, autocorrelation, and random decrement signatures are methods for measuring the response of a system to unknown random disturbances (such as aerodynamic turbulence) in which averaging is used to control the error level. Such methods require a large amount of data to extract accurately the information from the response signal. The test time needed to collect the data is often impractically long, particularly for full-scale aircraft (which have low fundamental structural frequencies). Moreover, when only the output is measured, there is always some uncertainty as to whether the system parameters are being estimated correctly. In some cases, the properties of the unknown input or the measuring system can influence the response measurement significantly (see Ref. 7 for a further discussion). By correlating the response and a measured external input as well as averaging, the test time for a given level of accuracy can be reduced significantly (by a factor of 10 to 15 in the example of Ref. 7). In fact, an estimate of the error level in the response is available directly from the measured data (see Refs. 8 and 9). Furthermore, the properties of the system alone are obtained in the form of the transfer function between the output and the input.

The present paper is concerned with the task of estimating the modal parameters from the system response measurement. To be useful in flutter testing, a technique must work well with noisy data and must be suitable for on-line stability measurements. An elementary algorithm for the frequency and damping of low-damped modes was developed in Ref. 7, based on integrating the transfer function in the vicinity of a resonant peak. This algorithm, however, is limited to a single-input, single-output transfer function

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measurement, and it was based on the response of a single-degree-of-freedom, second-order system; hence it generally was limited to analyzing the fairly low-damped modes. That is the most important information for flutter testing, but still the algorithm is only approximate, and the information obtained about the system is not complete. Gupta has derived an instrumental variables algorithm for dynamic systems that uses multi-input, multi-output data to identify a complete modal representation.[10] Basically this algorithm fits a set of poles and zeros to the measured transfer function. An instrumental variables estimate is similar to the least-squared-error estimate but without the bias of the latter for noisy data.[11] In the following sections, a frequency-domain derivation of the instrumental variables algorithm is given, and an example of its application to aeroelasticity testing is presented.

System Definition

Consider a linear, time-invariant dynamic system of order n. Such a system can be described by a set of linear constant-coefficient differential equations of the form

$$y^{(n)} + \sum_{k=1}^{n} a_k y^{(n-k)} = \sum_{k=1}^{m} b_k u^{(m-k)} + \text{noise}$$

where y is the vector of output measurements (dimension p), and u is the measurable input exciting the system. (Only a single input is considered here.) The vectors of the parameters to be identified are

$$\alpha^T = -(a_1,\ldots,a_n)$$

$$\beta_j^T = (\beta_{j0},\ldots,\beta_{jm}) = j\text{th row of } [b_0,\ldots,b_m]$$

for $j=1$ to p. Equivalently, this system is described by the transfer function

$$H_j = \frac{\beta_{j0}s^m + \beta_{j1}s^{m-1} + \ldots + \beta_{j(m-1)}s + \beta_{jm}}{s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n}$$

where s is the Laplace variable. The poles are the roots of the denominator polynomial, the zeros of y_j/u are the roots of the numerator polynomial. It has been assumed that the system can be described by n poles ($n \geq 1$) and m zeros ($m \geq 0$).

Instrumental Variables

A system input/output relation can be formulated as $y = X\theta + v$, where y and X are the measured data, θ is the vector of system parameters to be identified, and v is random noise with zero mean. The least-squared-error estimate of θ is $\hat{\theta}_{LS} = (X^TX)^{-1}X^Ty$. If the measurements X and the noise v are uncorrelated, this estimate is accurate. The expected value is, however, $E(\hat{\theta}_{LS}) = \theta + (X^TX)^{-1}E(X^Tv)$, and so, if X and v are correlated, the least-squared-error estimate will have a bias. In the presence of noise, least-squared-error techniques do not give useful results. In the instrumental variables technique, a matrix Z is selected which is not correlated with v. Then the instrumental variables estimate $\hat{\theta}_{IV} = (Z^TX)^{-1} Z^Ty$ has no bias, $E(\hat{\theta}_{IV}) = \theta$. In the present case, we use for instrumental variables the measurements at a past time. The time lag τ between the current measurements and the instrumental variables must be long enough so that the current random noise is not correlated with the measurements at τ. Averaging all of the data to obtain an input/output relation of the form $y = X\theta$ leads to auto- and cross correlations. Equivalently, we can work in the frequency domain and use integrals of the spectra. Note that, if the parameter τ is set to zero, the algorithm reduces to a least-squared-error estimate.

Instrumental Variables Algorithm for Flutter Testing

The differential equation describing the dynamic system can be written

$$y^{(n)} = [y^{(n-1)}\ldots y]\alpha + \begin{bmatrix} \beta_0^T \\ \vdots \\ \beta_p^T \end{bmatrix} \begin{pmatrix} u^{(m)} \\ \vdots \\ u \end{pmatrix}$$

Premultiplying this equation by

$$[y^{(n-1)}\ldots y]^T |_{t-\tau}$$

and taking the expected value gives

$$B_0 = A_{11}\alpha + \sum_{j=1}^{p} A_{12_j}\beta_j + E[(y^{(n-1)}\ldots y)^T |_{t-\tau} v]$$

where the matrix coefficients are obtained from the derivatives of the auto- and cross-correlation functions evaluated at τ (see Ref. 10), and v is the equation noise. This is a set of linear algebraic equations for α and β_j if the last term can be dropped, which is the case if τ is selected such that the outputs (and their time derivatives) at a time $(t-\tau)$ in the past have negligible correlation with the equation noise at the current time t. Similarly, multiplying each of the $y_1,\ldots,y_p$ equations by $(u^{(m)}\ldots u)^T$ and taking the expected values gives algebraic equations of the form

$$B_j = A_{21_j}\alpha + A_{22}\beta_j$$

No time lag is required in this case, since

$$E[(u^{(m)}\ldots u)^T v^T] = 0$$

for all the time instants. (The equation noise is not correlated with the input.) Considering a sampled-data or discrete system with sampling interval Δt, Ref. 10 obtains $2n\Delta t$ as the minimum value of the instrumental variables time lag τ (see also the discussion in the paragraphs below). Note that another set of equations for α and β_j may be derived by taking a delay twice the minimum value (and a third set using a delay of 3τ, etc.). This would provide an independent estimate of the parameters of the transfer function, which could be used to check for linearity and stationarity or could be combined with previous estimates to reduce errors. Additional details of the time domain derivation of the algorithm are given in Ref. 10.

Frequency Domain Derivation

A frequency domain derivation is more appropriate to the manner in which the algorithm is implemented here. In the frequency domain, the equations of motion are

$$(i\omega)^n Y = Y((i\omega)^{n-1}\ldots 1)\alpha + \begin{bmatrix} \beta_0^T \\ \vdots \\ \beta_p^T \end{bmatrix} \begin{pmatrix} (i\omega)^m \\ \vdots \\ 1 \end{pmatrix} U$$

where Y and U are the Fourier transforms of y and u. Now premultiply this equation by $[(-i\omega)^{n-1}\ldots 1]^T\ Y^{T^*}e^{i\omega\tau}$ and each of the $Y_1,\ldots,Y_p$ equations by $[(-i\omega)^m\ldots 1]^T U^*$. Note that the auto- and cross spectra are $S_{uu} = U^*U$, $S_{y_iy_i} = Y_i^*Y_i$, and $S_{uy_i} = U^*Y_i$ (where an asterisk denotes the complex conjugate). We also define

$$S_y = Y^{T^*}Y = \sum_{i=1}^{p} S_{y_iy_i}$$

The following set of equations is obtained:

$$\begin{bmatrix} i\omega^{2n-1} \\ \vdots \\ i^n\omega^n \end{bmatrix} S_y e^{i\omega\tau} = S_y e^{i\omega\tau} \begin{bmatrix} \omega^{2n-2} & \dots & i^{-n+1}\omega^{n-1} \\ \vdots & & \vdots \\ i^{n-1}\omega^{n-1} & \dots & 1 \end{bmatrix} \alpha$$

$$+ \sum_{j=1}^{p} S^*_{uy_j} e^{i\omega\tau} \begin{bmatrix} i^{m-n+1}\omega^{n+m-1} & \dots & i^{-n+1}\omega^{n-1} \\ \vdots & & \vdots \\ i^m\omega^m & \dots & 1 \end{bmatrix} \beta_j$$

and, for $j = 1$ to p,

$$\begin{bmatrix} i^{n-m}\omega^{n+m} \\ \vdots \\ i^n\omega^n \end{bmatrix} S_{uy_j} = S_{uy_j} \begin{bmatrix} i^{n-1-m}\omega^{n+m-1} & \dots & i^{-m}\omega^m \\ \vdots & & \vdots \\ i^{n-1}\omega^{n-1} & \dots & 1 \end{bmatrix} \alpha$$

$$+ S_{uu} \begin{bmatrix} \omega^{2m} & \dots & i^{-m}\omega^m \\ \vdots & & \vdots \\ i^m\omega^m & \dots & 1 \end{bmatrix} \beta_j$$

Each coefficient is now integrated over the desired frequency range ($\omega = -\omega_1$ to $-\omega_0$, and $\omega = \omega_0$ to ω_1). The result is a set of linear algebraic equations for α and β_j.

$$B_0 = A_{11}\alpha + \sum_{j=1}^{p} A_{12_j}\beta_j$$

$$B_j = A_{21_j}\alpha + A_{22}\beta_j$$

Solving the second set gives $\beta_j = A_{22}^{-1}(B_j - A_{21_j}\alpha)$, and so

$$\left(A_{11} - \sum_{j=1}^{p} A_{12_j}A_{22}^{-1}A_{21_j}\right)\alpha = B_0 - \sum_{j=1}^{p} A_{12_j}A_{22}^{-1}B_j$$

or $C\alpha = d$. The solution is $\alpha = C^{-1}d$, from which β_j can be calculated as well.

The coefficients of the matrices are as follows:

$$(B_0)_k = (-1)^{(1+k)/2} D_{2n-k}$$

$$(A_{11})_{k\ell} = (-1)^{(1+k-\ell)/2} D_{2n-k-\ell}$$

$$(A_{12_j})_{k\ell} = (-1)^{(2+m-n+k-\ell)/2} E_{n+m-k-\ell+1}$$

$$(B_j)_k = (-1)^{(n-m+k)/2} F_{n+m-k+1}$$

$$(A_{21_j})_{k\ell} = (-1)^{(n-m+k-\ell)/2} F_{n+m-k-\ell+1}$$

$$(A_{22})_{k\ell} = (-1)^{(1+k-\ell)/2} G_{2m-k-\ell+2}$$

where the integer truncation is used to evaluate the exponent of the powers of -1. The row index k and the column index ℓ count from 1 at the upper left-hand corner of a matrix. The required integrals of the spectra are

$$D_k = \begin{cases} 2\int_{\omega_0}^{\omega_1} \omega^k S_y \cos\omega\tau \, d\omega; & k \text{ even} \\ 2\int_{\omega_0}^{\omega_1} \omega^k S_y \sin\omega\tau \, d\omega; & k \text{ odd} \end{cases}$$

$$E_k = \begin{cases} 2\int_{\omega_0}^{\omega_1} \omega^k (ReS_{uy_j}\cos\omega\tau + ImS_{uy_j}\sin\omega\tau) \, d\omega; & k \text{ even} \\ 2\int_{\omega_0}^{\omega_1} \omega^k (ReS_{uy_j}\sin\omega\tau - ImS_{uy_j}\cos\omega\tau) \, d\omega; & k \text{ odd} \end{cases}$$

$$F_k = \begin{cases} 2\int_{\omega_0}^{\omega_1} \omega^k ReS_{uy_j} \, d\omega; & k \text{ even} \\ 2\int_{\omega_0}^{\omega_1} \omega^k ImS_{uy_j} \, d\omega; & k \text{ odd} \end{cases}$$

$$G_k = \begin{cases} 2\int_{\omega_0}^{\omega_1} \omega^k S_{uu} \, d\omega; & k \text{ even} \\ 0; & k \text{ odd} \end{cases}$$

It is simplest to determine the sign of the coefficients by using the pattern established in the matrix, which is the same relative to the lower right-hand corner for all of the matrices.

The spectra should be scaled by dividing by the input autospectrum S_{uu} to eliminate spurious peaks. In that case, the algorithm works with integrals of the transfer function. To avoid numerical difficulties associated with the powers of ω, the frequency should be scaled by dividing by the maximum frequency in the spectrum, here the upper limit of integration ω_1. The poles and zeros then also are scaled with ω_1; and the normalized instrumental variables time lag is $2\pi\omega_1\tau$, with ω_1 in Hertz. The various outputs y_i can be weighted as desired to emphasize the measurements known to be of highest quality. It should be noted that the original measurements are likely to be in volts, with about the same amplitude for all signals. Hence using engineering units is equivalent to weighting the measurements. All of the signals could be scaled with their rms values to eliminate such spurious weighting.

With the algorithm in the frequency domain, all spectra can be put through the same filter in order to improve the identification. Two such filters already have been included in dividing the spectra by the input autospectrum S_{uu} and limiting the integration to the range ω_0 to ω_1. In a similar fashion, discrete frequency noise in the data can be eliminated by dropping the appropriate spectral lines from the integrals. Normally the zero frequency line would be omitted as well. The frequency range ω_0 to ω_1 should correspond to the input signal bandwidth. If too small a range is used, the identified parameters will lose physical relevance.

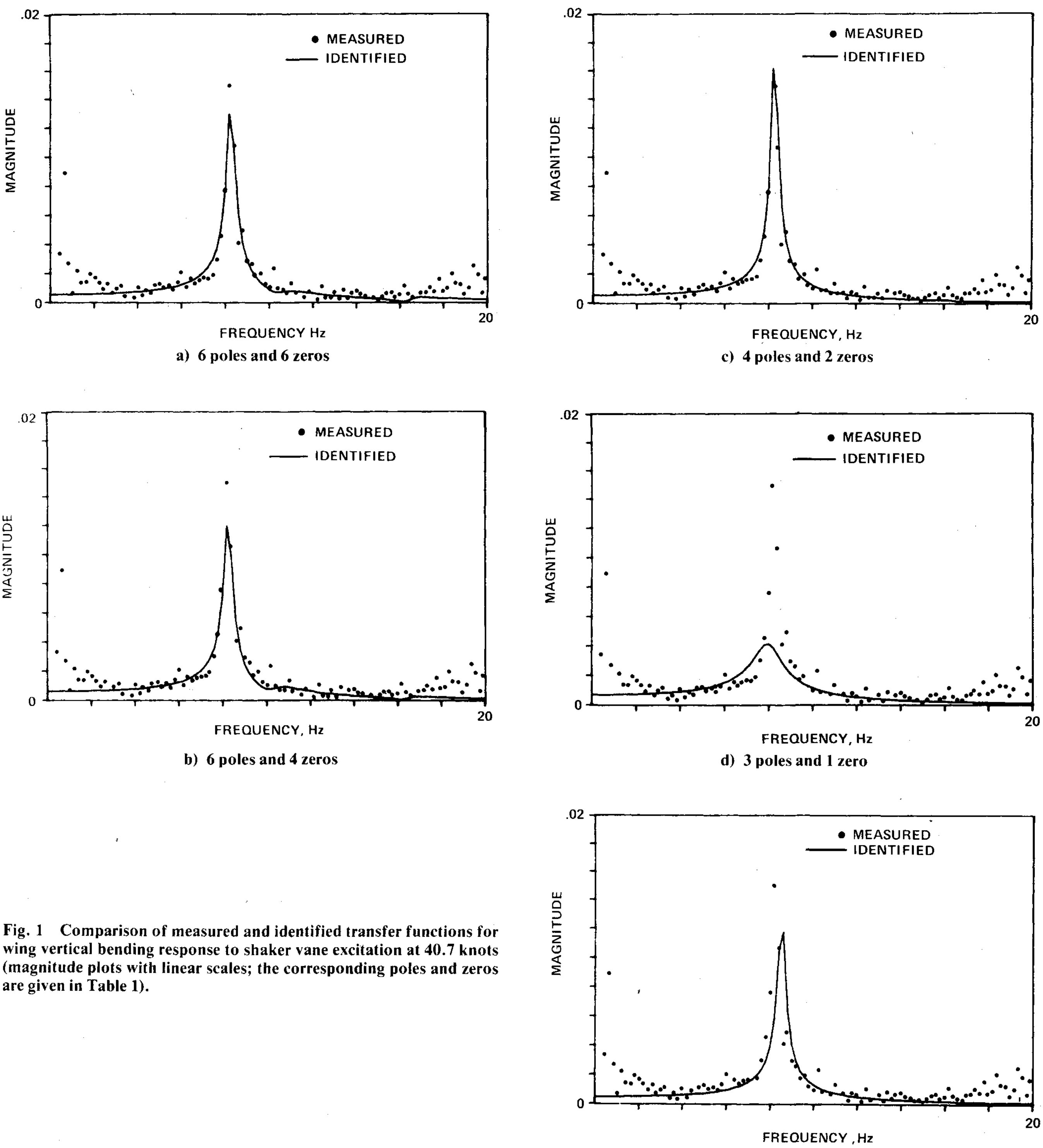
The bias of the parameter estimates due to noise is eliminated by the proper choice of the time lag τ. With $\tau = 0$, a least-squared-error estimate is obtained. For an instrumental variables estimate, τ must be greater than the time required for the effects of the noise to propagate between the current measurements and the instrumental variables. This propagation time depends on the order of the equation assumed to represent the system. Considering a sampled-data or discrete system, Ref. 10 derives for the propagation time $2n\Delta t$, where Δt is the sampling interval and n is the order of the system. The maximum frequency in a time series is $(2\Delta t)^{-1}$. Hence the instrumental variables time lag should be $\tau = n/\omega_{\max}$, where $\omega_{\max}$ is the maximum frequency present in the spectra (in Hertz). With the upper limit of the integration at ω_1, then, $\tau = n/\omega_1$ should be used.

The extension of the algorithm to the multi-input case is given in Ref. 10. A test involving several simultaneous, independent inputs is not recommended, however. A significant reduction in the input levels, compared to what could be achieved with individual inputs, probably would be required to stay within the limitations placed on the response. Hence the signal-to-noise ratio would be reduced. Also, additional averaging would be required to obtain the same error level in the transfer functions as with a single input. Therefore, simultaneous multiple inputs perhaps could require more total test time to achieve a given accuracy than using successive single inputs. Perhaps most important, the use of several simultaneous inputs would greatly increase the complexity of the test procedure. The multi-output form of the algorithm is probably best suited for post-test data reduction, when it is possible to try various weighting combinations to obtain the most accurate parameter estimates. Thus, in its present stage of development, the single-input, single-output form of the instrumental variables algorithm is most useful for on-line estimation of stability in flutter testing.

This instrumental variables algorithm was applied in digital computer simulations of several dynamic systems in Ref. 10. These simulations examined the influence of the choice of τ; of random and discrete noise; of the number of outputs measured; of the data length; and of the model order assumed. Systems with up to 13 degrees of freedom were considered. It was verified that, with the theoretical value of τ, the bias in the estimate due to noise is eliminated.

Example of Application

To demonstrate the application of this instrumental variables algorithm, it was used to analyze data from a flutter test of a proprotor and cantilever wing model in a wind tunnel.[12] The three-bladed, gimbaled stiff in-plane rotor had a diameter of 0.86 m and was operated at a rotational speed of 22.67 Hz. The model was tested in the Massachusetts Institute of Technology Wright Brothers Wind Tunnel, over a speed range of approximately 40 to 80 knots with the rotor in axial flow. The rotor controls consisted of collective and cyclic pitch actuators. The pylon also had a shaker vane that was equivalent in action to a wing flaperon. The fundamental



a) 6 poles and 6 zeros

b) 6 poles and 4 zeros

c) 4 poles and 2 zeros

d) 3 poles and 1 zero

e) 2 poles and no zeros

Fig. 1 Comparison of measured and identified transfer functions for wing vertical bending response to shaker vane excitation at 40.7 knots (magnitude plots with linear scales; the corresponding poles and zeros are given in Table 1).

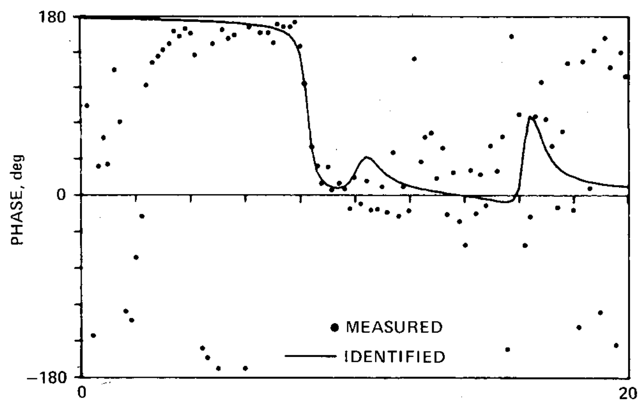
Table 1 Identified roots for wing vertical bending response to shaker vane at 40.7 knots ($\omega_0 = 4$ Hz, $\omega_l = 17$ Hz)

n/m	Identified poles p and zeros z, Hz	
6/6	$p = -0.175 \pm i8.26$	$p = -0.510 \pm i16.60$, $-0.682 \pm i10.43$ $z = -0.095 \pm i16.17$, $-0.423 \pm i10.30$, -22.76, -26.48
6/4	$p = -0.175 \pm i8.26$	$p = -0.510 \pm i16.61$, $-0.656 \pm i10.46$ $z = -0.166 \pm i16.18$, $-0.451 \pm i10.14$
4/2	$p = -0.131 \pm i8.27$	$p = -0.274 \pm i16.46$ $z = -0.267 \pm i16.52$
3/1	$p = -0.672 \pm i7.99$	$p = 29.05$ $z = 41.67$
2/0	$p = 0.162 \pm i8.52$	

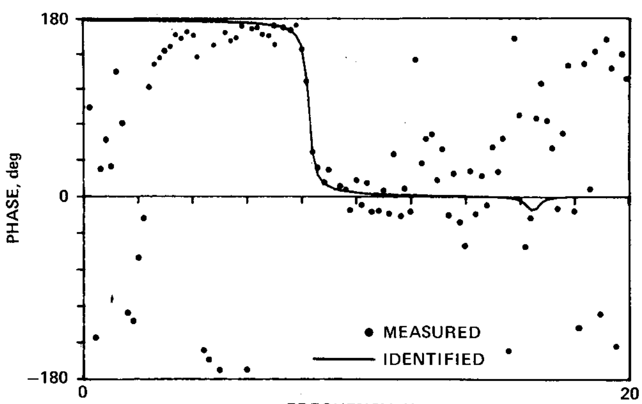
wing frequencies were approximately 8.2 Hz for vertical bending and 12.5 Hz for chordwise bending. Reference 12 gives complete details of the model. The shaker vane was used to excite the wing vertical bending mode, and rotor collective pitch was used to excite the wing chordwise bending mode. The input signal was white noise sent through a 3- to 12-Hz bandpass filter. The control actuators had essentially flat frequency response over this range (see Ref. 7). The shaker vane rms level used to excite the system was 1.5 to 2.0 deg, and the rms collective pitch excitation was 0.25 to 0.75 deg. The one-half peak-to-peak level was around three or four times the rms value. The input level used was determined by the requirements for a significant response level and the limitations on blade and wing loads. The resulting wing vertical and chordwise bending total response was two or three times the response due to the existing tunnel turbulence alone. The model was tested by successively exciting the shaker vane and rotor collective pitch controls at four air speeds.

The experimental procedure for measuring the system transfer function are discussed in detail in Ref. 7. With digital data reduction, the parameters defining the transfer function measurements are the sample rate r, the sample size N, and the number of averages K. The maximum frequency in the spectra is the Nyquist frequency $\omega_{max} = r/2$ Hz, and so the sample rate is chosen on the basis of the signal bandwidth. The sample rate should be at least 2.5 times the bandwidth of interest and about 4 times the frequency of the model to be analyzed. (Data are suspect near the maximum frequency because of aliasing and near zero frequency because of bias error.) The choice of the number of samples N determines the frequency resolution $\Delta\omega = r/N$ Hz. To avoid significant bias error in the transfer function, $\Delta\omega \cong \zeta\omega_n$ is required (where ζ is the damping ratio of a resonant peak, and ω_n is the natural frequency; see Refs. 7 and 8), giving $N = 256$ or 512 typically. With multimode systems, $\Delta\omega$ also will have to be small enough to discriminate any close poles adequately. In practice, the required sample size is established by trying two or three values at the beginning of the test, until a value of N is found which gives no noticeable bias error. (The amplitude of the transfer function at resonant peaks is the primary criterion.) The number of averages K determines the random error level in the transfer function measurement. The total test time is $T = K/\Delta\omega$, and so normally the number of averages used is a compromise between an acceptable test time and what one would like for very small error. For the present case, a sample rate of $r = 51.2$/s was used, with $N = 256$ samples collected for a single record, giving a frequency resolution of $\Delta\omega = 0.2$ Hz. Ten averages normally were used, for a total data length of 50 s.

The parameters required by the instrumental variables algorithm are the number of poles n, the number of zeros m, and the frequency range of integration ω_0 to ω_l. For the present example $\omega_0 = 4$ Hz and $\omega_l = 17$ Hz were chosen as covering the bandwidth of the input signal and including both the vertical and chordwise bending modes. The model order appropriate to the system is established at the beginning of a test by trying several values of m and n with the first set of data. The criteria for the choice of n and m are a good fit of the measured transfer function and the best estimate of the frequency and damping. A direct comparison of the identified and measured transfer functions was found to be very valuable in establishing the appropriate model order. Table 1 and Fig. 1 illustrate the procedure, for the case of wing vertical bending response to shaker vane excitation at 40.7 knots. Table 1 gives the poles and zeros identified for a number of combinations of n and m, and Fig. 1 compares the measured and identified transfer functions. Examining the measured transfer function, it is concluded that the system is basically second order, with a natural frequency around 8 Hz. For the first attempt at identification, a clearly overspecified model order is chosen, in this case 6 poles and 6 zeros. Table 1



a) 6 poles and 4 zeros



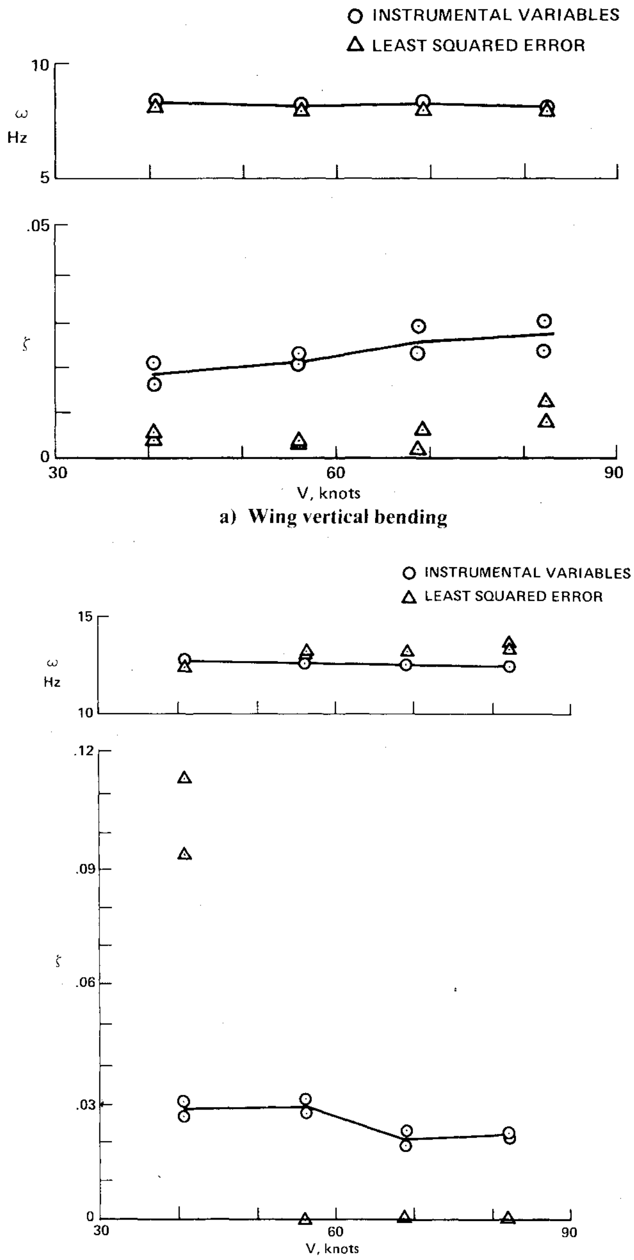
b) 4 poles and 2 zeros

Fig. 2 Phase of measured and identified transfer functions for wing vertical bending response to shaker vane excitation at 40.7 knots (the corresponding poles and zeros are given in Table 1).

shows that the pole at 8.26 Hz is identified, and the remaining roots occur in nearly canceling pole-zero pairs or are outside the frequency range considered, a characteristic result when the assumed model order is higher than the true order of the system being tested. In such a case, it is necessary to rely on experience with the algorithm and with the system being tested to discriminate between the spurious and the true roots. Noting that two zeros are well outside the range $\omega_0 = 4$ to $\omega_l = 17$ Hz with $n = m = 6$, 6 poles and 4 zeros are tried. Considering the remaining nearly coincident pole-zero pairs, $n = 4$ and $m = 2$ is tried, followed by $n = 3$ and $m = 1$, and finally $n = 2$ and $m = 0$. Based on the fit of the identified transfer function to the measured data and the behavior of the identified pole around 8 Hz, it is concluded that the appropriate model order in this case is 4 poles and 2 zeros. Figure 2 shows the corresponding phase of the measured and identified transfer functions. By a similar procedure, $n = 4$ and $m = 2$ was determined to give the best results for the wing chordwise bending response to collective excitation as well.

Proper choice of the model order is evidently important for good estimates of the modal parameters. Using a lower order than $n = 4$ and $m = 2$ gives poor results in the present example, even though the system is basically second order. The additional pole-zero pair are close and near the boundary of the frequency range considered and hence are disregarded readily. This pair of roots is necessary, however, for a good overall fit of the measured transfer function, allowing a better estimate of the true root to be obtained from the remaining pole. The parameter estimates also degrade as the model order is increased, although the estimate of the principal root is much less sensitive to overspecification of the model than it is to underspecification. As the model order is increased, the identified transfer function first becomes lumpy because of the spurious poles and zeros (see Figs. 1b and 2a). Eventually, as the model order increases, the matrices in the instrumental variables algorithm become rank deficient, and numerical difficulties result. As the flutter test progresses, the model order can be monitored by checking the fit of the identified transfer functions to the measured data.

Figure 3 presents the frequency and damping ratio of the wing vertical and chordwise bending modes, as identified using the instrumental variables algorithm. Figure 4 compares the modal parameter estimates of the instrumental variables algorithm with least-squared-error estimates (obtained by setting the instrumental variables time lag τ to zero). The bias due to noise in the least-squared-error procedure results in useless estimates; the damping estimate can be far off, either high or low, and there can even be a large bias in the frequency estimate (Fig. 4b). The scatter of the damping ratio estimate shown in Fig. 3 is due to the small number of averages used ($K = 10$), which results in a high noise level in the measured transfer function (Fig. 1). The data also were analyzed with $K = 20$ for less noise or $N = 512$ for less bias, but with the instrumental variables algorithm there was generally little difference in the identified roots. Indeed, with $K = 5$ or $N = 128$ there was more scatter, but still consistent identification of the stability was obtained. The instrumental variables algorithm is not sensitive to noise in the transfer function measurement, and so a small number of averages can be used. Moreover, it uses all of the data and thus reduces the effect of any bias at the resonant peak, allowing the use of a larger frequency resolution. Hence the total data length $T = K/\Delta\omega$ is minimized by use of the instrumental variables algorithm. With a less powerful data analysis technique, it would be necessary to increase the test time substantially in



a) Wing vertical bending

b) Wing chordwise bending

Fig. 4 Comparison of frequency and damping ratio identified by instrumental variables and least-squared-error algorithms.

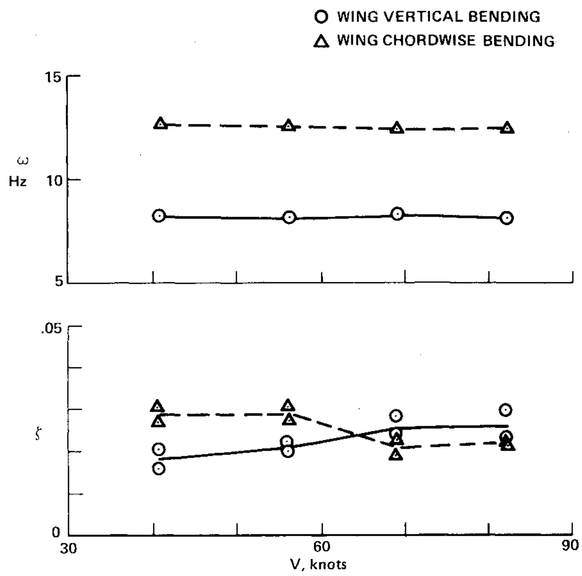


Fig. 3 Frequency and damping ratio of the wing vertical and chordwise bending modes identified by the instrumental variables algorithm.

order to estimate the modal parameters with confidence. With the instrumental variables algorithm, it always is desirable to increase K and N in order to reduce the scatter in the estimates, but the technique allows accurate parameter identification even with the small number of averages and small number of samples often required for an acceptable test time.

Although the present experimental data do not allow a demonstration, the instrumental variables algorithm can identify the parameters of multimode systems, including the ubiquitous case (in theory if not in practice) of two very close low-damped modes. We have applied the algorithm to numerous arbitrarily constructed transfer functions and found that, even for the case of several close poles, the modal parameters are identified exactly if no noise is included. The most important consideration in such cases is that the frequency resolution $\Delta\omega$ be small enough to discriminate the modes.

An elementary algorithm for the frequency and damping of low-damped modes, based on integrating the transfer function in the vicinity of a resonant peak, was developed in Ref. 7. The advantage of that technique is that it can be implemented on analysis systems with limited computational capability. Compared with the instrumental variables results, it was found that consistent estimates of the damping were obtained using this resonant peak analysis, but that about twice the number of averages K were required for a comparable scatter in the identified parameters.

The computer core and computation time requirements of the instrumental variables algorithm were a concern, but no difficulties were encountered in the present application. The algorithm was programmed in FORTRAN under the disk-based RT-11 operating system, for a DEC PDP 11/45 computer with 32K of memory. An overlay structure was used, with one segment for the data acquisition subprogram, one for the instrumental variables subprogram, and others for plot, list, and utility routines. Generally the core required for the instrumental variables program was comparable to that required for data acquisition, and the computation times were considerably smaller. The core and computation time requirements of the instrumental variables algorithm depend on the number of lines in the spectra and on the model order. The algorithm was programmed for maximum values of $N=1024$, $n=20$, and $m=20$. For the present example, with $N=256$ the computation time was about n s (where n is the model order). Data collection alone required 50 s.

Concluding Remarks

An instrumental variables algorithm for modal parameter identification has been derived in the frequency domain, and an example of its use in aeroelasticity testing has been given. It has been demonstrated that this technique can be implemented for on-line data reduction with a minicomputer-based analysis system. This parameter identification algorithm makes no assumptions about the structure of the system except that it is linear and time-invariant, basically fitting a set of poles and zeros to the measured transfer function. By using instrumental variables, the sensitivity of the modal parameter estimates to noise in the response measurements is reduced greatly. Hence accurate and reliable stability estimates can be obtained from a resonable length of data. The algorithm is expected to be a powerful and valuable tool for on-line estimation of modal parameters in flutter testing and also should be useful in control system and structural dynamics tests.

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